

## LANGUAGE STRATEGIES FOR MATHEMATICS

Included here are Strategies for integrating language and literacy learning into high school math, as compiled research by **Stanford University**

[http://suse-step.stanford.edu/resources/LanguageSite/Math\\_Strategies.html](http://suse-step.stanford.edu/resources/LanguageSite/Math_Strategies.html)

### Using Graphic Organizers to Improve the Reading of Mathematics

This strategy uses a graphic organizer to improve mathematics reading comprehension, verbal communication, writing, and story problem solving skills. It leads students through the 5 steps:

- Restate the problem question in the student's own words.
- Identify information required to solve the problem, including data appearing directly in the problem or data that may need to be computed.
- Plan the mathematical calculations that must be performed, including the appropriate sequence.
- Perform the required calculations.
- Review work to confirm that the solution is accurate and reasonable

Lind argues for adding what he believes to be a 6th essential step:

- Explain and justify your solution to your classmates.

\*See a sample graphic organizer.

Lind emphasizes several literacy benefits from this strategy:

- In reading for comprehension, students must conceptualize the situation and express it in their own words, thinking deeply to effectively organize their writing.
- Students must filter through all the information and determine which data are relevant and which additional data are required.
- It can be used at first for small group tasks and then as a basis for whole-group discussion in a structured framework where students clarify and justify their approach with other classmates., encourage thorough and persuasive oral communication.
- It can be adapted for more complex, multi-stage problems by replacing the diamond format with an open note format, preserving the sequencing of the steps.

Lind suggests introducing the organizer with an over-

head transparency, describing each step/section while asking the class to perform each step for a given problem. After multiple uses of the organizer, he recommends gradually releasing responsibility of completing it to the class and states that this strategy works especially well for group activities where each team member can facilitate and record the group's response to a given section. It also can provide a great way to compare and contrast alternative approaches for solving the same problem across teams and to discuss in which circumstances one approach may be better than another.

### Encouraging "Math Talk" in the Classroom

This strategy focuses on creating a safe classroom environment that enables students to become more proficient and comfortable with discussing mathematics and justifying their ideas which in turn helps them clarify and deepen their concept understanding, recognize and learn alternative ways to solve the same problem, and develop confidence in their ability to think mathematically. Encouragement of "math talk" comes through:

- Establishing norms which promote open, interactive discussions including:
  - Respecting the opinions of others (no put downs); teacher vehemently condemns any negative comments toward an individual or an idea upon first infraction; reinforce respect by noting whenever possible, positive aspects of student's thinking, even if solution was wrong.
- Celebrating failures, risk taking; Acknowledge individual Misunderstandings as great opportunities for whole class learning. Praise especially creative ways at attempting to solve a problem.
- Demanding critical thinking; teacher models by thinking out loud as he/she reasons through alternative solutions. Teacher asks questions that prompt students to be rigorous in their explanations and proofs and praise deep analysis even if the answer turns out wrong.
- Asking open-ended questions that require true communication among students (not 1 or 2 word responses). "How did you arrive at that conclusion?" "Why do you think this answer is correct?" "Does everyone agree with this answer? Why or why not?" "Did anyone get the same answer using a different approach?"
- Shifting responsibility onto the class for explaining mathematical concepts and solutions. Throw a student's question or idea back to the class for response.

Lind points out that these behaviors are not natural for middle and high school adolescents or anyone else and

so require much perseverance, modeling and coaching. Learning these behaviors is well worth it, even though it may significantly change the curriculum balance in terms of quantity versus quality.

### **Assessing Students' Beliefs about Mathematics**

The strategy is to have students discuss open-ended questions regarding beliefs/biases about mathematics. Spangler proposes discussion questions that point to commonly held misconceptions about mathematics because, "Students' learning experiences are likely to contribute to their beliefs about what it means to learn mathematics. In turn, their beliefs about mathematics are likely to influence how they approach new mathematical experiences." The questions can be presented in written homework, journal assignments, research/interview topic. Some of the key questions follow:

- What subject(s) in mathematics do you most like? Least like? Why?
- Is it possible to get the right answer to a mathematics problem and still not understand the problem? Explain.
- If given a choice, when solving a problem would you prefer to have one method that works all the time or many methods that work all the time?
- If you and a friend got different answers to the same problem, what would you do?
- Do you suppose McDonald's has a mathematician on its corporate staff? What might that person do for McDonald's?

Joss advocates using this strategy at the beginning of the year because the most important first step in establishing a culture in a group is to get the crucial biases and realities and how the class will operate out in the open early on. Joss proposes to use the author's questions in a think-pair-share type of discussion with capturing of ideas on the board and with obtaining responses from other students. He would revisit some of the ideas later in the year in different forms to assess how attitudes may be changing.

### **Making Mathematical Arguments**

Having students orally justify and explain their ideas supports and models the process of mathematical thinking and infuses the class with norms that 'questioning is a sign of good thinking' and that 'being able to justify one's ideas is valuable'.

Talk about mathematics teaches that there can be multiple methods for problem solving and makes visible the

link between personal reasoning and shared meaning in the classroom.

There are many ways to apply this strategy at different levels in the classroom-- large group discussion, collaborative groups or dyads may be appropriate for different problem solving tasks.

Barnett suggests giving students a challenging multi-part problem and asking them to share their ideas about how to solve it, rather than focusing on the solutions. This may help to create an environment where the process of math is valued as much as the outcome. By presenting their thinking, students will have ongoing opportunities to develop their oral communication skills, and are likely to benefit as well from exposure to the diversity of their classmates' ways of thinking and solving problems.

### **Sort and Predict**

This is a pre-reading, pre-instructional activity where students look for similarities and differences in word structure (or in shapes or illustrations) and group them accordingly. In an introductory geometry lesson students might sort, group and predict meanings, then read or listen to related information and finally participate in a whole group discussion, checking their predictions and revising them into definitions.

Lind points out that once the student recognizes and learns some of the key patterns and roots in math terms, it is easier to figure out meanings of new words and to memorize and recall them. Lind would use this strategy for math lessons that introduce a large number of new vocabulary words and he would have students create posters of sorted words, predictions and definitions to post as references. He suggests pairing the words with their equivalents in other languages to support EL students' learning. He also recommends having students make pictures to establish visual associations for terms.

### **Using Communication to Develop Math Literacy**

To encourage communication in the math classroom as a strategy for improving oral literacy and mathematical thinking, teachers must first attend to creating a sense of classroom community through mechanisms of carefully chosen oral feedback, established norms for responding to one another and for accepting criticism. Learning to craft and pose questions that make students think is the key to teachers' and peers' ability to stimulate real progress toward math literacy. When engaging in communication about their math learning, activities should be structured so that students gain balanced experience in the following domains:

- Organize and consolidate their mathematical thinking through communication

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- Communicate their mathematical thinking clearly to peers, teachers and others
- Analyze and evaluate their mathematical thinking and strategies of others
- Develop facility with mathematical concepts by examining the methods and ideas that others used, to determine relative strengths and limitations in those approaches

### Writing About the Problem-Solving Process

Students are asked to write about how they applied Polya's four-step "executive process" as they solve a problem. The four steps include:

- Understanding what the problem is asking for
- Devising a strategy for solving the problem
- Carrying out the plan
- Looking back, reflecting and seeing if the answer makes sense

With scaffolding and tailoring the reflection to the students' abilities, this training would be very helpful even before middle school.

### The Oral Quiz

Once or twice per grading period, students receive an Oral Quiz paper upon entering the classroom. This quiz includes points for being on time, prepared with materials and having completed homework and good behavior. The final question reads "I answered a question when called upon to do so." The teacher circulates in the room to confirm students' answers to the concrete questions, and asks the group as many questions as it takes to make sure everyone has the chance to answer.

Degree of difficulty of the questions can vary, in order to meet student needs. Students may respond orally or write on the board. After the first question is answered, students may earn one point per question up to 105 points. The purpose of this is to stimulate all students to participate in oral discourse.

Grades are used in order to increase motivation for disengaged students. Because it can be anxiety-producing for students who are uncomfortable speaking in groups, this strategy should be applied with great care. The purpose may be more easily reached through variations with motives other than grades. More frequent use of the strategy may help students get comfortable with the pace and feel of the discourse in a lower-risk situation. Try emphasizing the thinking process and not just the answer; make sure you and your students clearly know what the aim of the activity is--to demonstrate knowl-

edge or to practice oral discourse?

### Socratic Seminars

With the objective of promoting understanding through inquiry rather than direct instruction, Socratic seminars are set up on the basis of the following guidelines:

- Respect one another's opinions
- Avoid interrupting, but there is no need to raise hands
- Address classmates by name
- Take notes
- Comments should address the topic and not digress
- Points of disagreement are settled among participants; the teacher is not a resource
- Sit in a circle so that everyone can be seen (or a semicircle at the board)

The teacher selects the topic and directs the discussion with skillful questioning that focuses the discussion, addresses inappropriate comments/behaviors, and clarifies key concepts that are raised. Use of this strategy promotes critical thinking, and can ultimately be more revealing of student thinking than other forms of assessment. Students have the opportunity to learn to revise, refine and improve their ideas; students who participated in a study of this method received higher scores on exams than those who did not, suggesting that knowledge gained can be detected with traditional measures.

Using thought provoking mathematical problems and questions is critical to the success of the seminar; as a teacher, it is essential to select tasks that make higher-level cognitive demands. Poon suggests tasks that involve "procedures with connections to understanding, meaning or concepts" and "doing mathematics that encourage students to explore multiple representations and solution methods.

Using Graphic Organizers (as recommended by James Barton) can help students make meaning and demonstrate effective note-taking. Establishing procedural norms with the group is also important for success; participation should be required, but thought process should be prioritized over "correctness" when it comes to grading. You can videotape seminars for deeper analysis of interactions/problem solving methods.

### Paired Reading

In Li and Nes' strategy, a skilled reader is paired with a less-skilled reader; dyads can include parent/child, teacher/student, student/student, volunteer/student. The

skilled reader models appropriate reading rate, inflection and pausing, reading aloud while the less-skilled reader follows along silently. The less-skilled reader then reads the same passage aloud. The skilled reader does not interrupt, offering help only if the less-skilled reader hesitates for a full 3 seconds, or if s/he asks. The text selected should be appropriate for the less-skilled reader's ability level, and passage length/difficulty should increase gradually. One-on-one modeling and feedback provides the ESL student with tools to increase fluency, a step on the road to reading enjoyment. Tutorial sessions before and after school are recommended

### The Frayer Model

Students can benefit from being guided to attend to words, and can develop more than an acquaintance with them by learning new words relationally. In the Frayer Model, students are given graphic organizer in which a page is divided into four quadrants, with a blank box at the center where the vocabulary word is to be written. The quadrants are labeled:

- Essential Characteristics
- Non-Essential Characteristics
- Examples
- Non-Examples

Students should be encouraged to use visual representations in addition to words as they establish understandings. This can be particularly useful for relating content matter like geometry in helping them SEE interrelationships. Students may also collaborate to make poster-size versions to illustrate key concepts on classroom walls. Poon adds that it is important for students to differentiate between geometric characteristics derived from definitions and those that are derived from theorems/corollaries; a simple coding system (D, T, C) may be used on the organizer to make this clear.

### Interactive Writing

Writing for math learning can help students to become conscious of their internal steps for problem-solving and aid the teacher in assessing how best to address students' needs by giving him/her a better sense of their backgrounds, strengths/weaknesses etc.

An in-class journal may also serve as a forum for direct student-teacher communication and relationship-building.

### Writing and Post-Writing Group Collaboration

Mathematical problem solving and writing involve many of the same thought processes (identifying issues, gathering information/resources, outlining ideas/argument, revising/evaluating solution etc.); engagement in problem solving can strengthen the thinking skills and processes that underlie good writing.

In the strategy developed by Johanning, students are presented with examples and a rubric for a quality written solution, and then asked write their solutions to a problem-solving prompt.

Students must explain the reasoning that led to their solution and support their selection of strategies with evidence. After drafting (and potentially a first revision of their writing) the students join a small group to share, listen and discuss their thoughts on possible solutions.

After group collaboration, students are encouraged to revise their written responses as appropriate. For this strategy to be successful, prompts must be selected from an appropriate 'zone' of understanding for the ability levels within the group.



### Using RIDGES to Solve Word Problems

1. **Read** the problem. If the problem is not understood, re-read it.
2. **Identify** all of the information given in the word problem. List the information separately. After listing all of the information, circle the information that is needed to solve the problem.
3. **Draw** a picture- Draw a picture of the information in the problem. This may help a student pick out the relevant information.
4. **Goal** Statements. The student should express, in his or her own words, the question the problem is asking.
5. **Equation** development- The student will write an equation to the problem. (i.e. length + width + length + width = distance around the field)
6. **Solve** the equation- The given information is plugged into the equation (i.e.  $10+6+10+6=\text{distance around the field}$ )

Source: Snyder, K. (1988) *Ridges: A problem-solving math strategy*. *Academic Therapy*, 230), 261-263