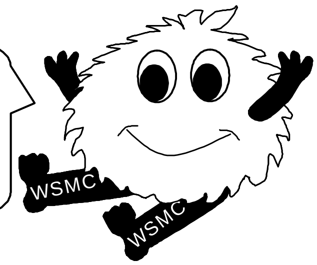
Math History

Got an idea for **Math History**?
Jot it down and send to Donna Buck,
PO Box 170, Joyce, WA 98343
(electronic information can be sent to
donna_buck@pasd.wednet.edu).



Imgainary Unit

= i, is the imaginary unit of any imaginary number. Discovered by the Italian mathematician Girolamo Cardano. An imaginary number is a number of the form **b*i*** where 'b' is a real number, '*i*' is the square root of -1, for b $\neq$ 0. Imaginary numbers (and complex numbers in general) are essential for describing physical reality and have concrete applications in: electromagnetism, signal processing, control theory, quantum mechanics, cryptology, and cartography… is the result of the following equations:

$x^2 + 1 = 0$

$x^3 - x = 0$ (for $x \neq 0$ or $x \neq 1$)

Square roots of negative numbers other than -1 can be written under the form:

$\sqrt{-n} = i\sqrt{n}$

$e^{i\pi/2} = \cos(\pi/2) + i \sin(\pi/2) = i$

$i^i = e^{-\pi/2} \approx 0.207879576...$

The reciprocal of i is $-i$: $i^{-1} = 1/i = i/i^2 = i/-1 = -i$

Powers of i repeat in a definite pattern (i, -1, $-i$, 1, ...):

$i^1 = i$

$i^2 = -1$

$i^3 = i^2 i = (-1)i = -i$

$i^4 = i^3 i = (-i)i = -(i^2) = -(-1) = 1$

$i^5 = i^4 i = (1)i = i$

Multiplicative table with *i*	**1**	**-1**	***i***	***-i***
1	1	-1	*i*	-*i*
-1	-1	1	-*i*	*i*
i	*i*	-*i*	-1	1
-i	-*i*	*i*	1	-1

The first roots of i are:

$\sqrt[1]{i} = i$

$\sqrt[2]{i} = \pm(1 + i)/\sqrt{2}$

$\sqrt[3]{i} = (\sqrt{3} + i)/2$

$\sqrt[4]{i} = \pm(i\sqrt{(2 - \sqrt{2})} + \sqrt{(2 + \sqrt{2})})/2$

$\sqrt[5]{i} = i$

A 'paradox' with i:

a) $\sqrt{-1} = \sqrt{-1}$

b) $\sqrt{(1/-1)} = \sqrt{(-1/1)}$

c) $\sqrt{1}/\sqrt{-1} = \sqrt{-1}/\sqrt{1}$

d) $(\sqrt{1})^2 = (\sqrt{-1})^2$

e) 1 = -1 and then 2 = 0 ??? Is this possible? Can you discover what led to this poetic licensed conclusion?

i to the i is a Real Number

If you are familiar with complex numbers, the "imaginary" number i has the property that the square of i is -1. It is a rather curious fact that i raised to the i-th power is actually a real number!

In fact, its value is approximately 0.20788.

Presentation Suggestions:
This makes a great exercise after learning the basics about complex numbers.

The Math Behind the Fact:
From Euler's formula, we know that exp(i*x) = cos(x) + i*sin(x), where "exp(z)" is the exponential function ez. Then

exp(i*Pi/2) = cos(Pi/2) + i*sin(Pi/2) = i.
Raising both sides to i-th power, we see that the right side is the desired quantity ii, while the left side becomes exp(i*i*Pi/2), or exp(-Pi/2), which is approximately .20788.

Su, Francis E., et al. "i to the i is a Real Number." Mudd Math Fun Facts. <http://www.math.hmc.edu/funfacts>.