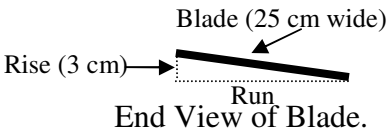
WSMC High School Regional Competition

The Ceiling Fan

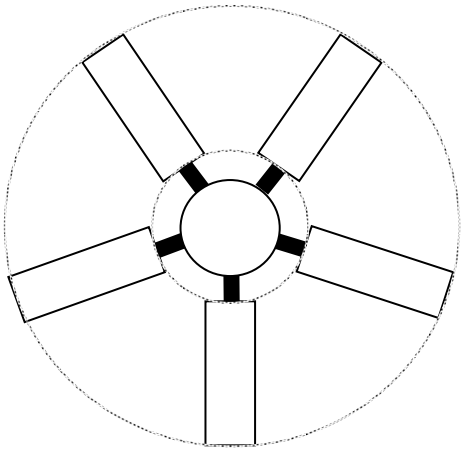
Team Problem

March 5, 2008

A ceiling fan is built to circulate the air in a room and is hung form the ceiling. Your goal is to vary the pitch of the blades to make the fan more efficient. The outer ends of the fan blades move at a greater speed than the inner ends thereby trying to move more air at the outer end than at the inner end. This problem is caused by the blades being flat (planar). Airplane and boat propellers solve this problem by varying the pitch of the blade (twisting the blade). Let's define the pitch of the blade as the rise divided by the run as shown in the end view of the blade.



End View of Blade.


Bottom View of the Fan

Consider a ceiling fan that has 25 cm wide rectangular planar blades. In the bottom view, the inner circle (the circle formed by the inner ends of the blades) has a diameter of 60 cm and the outer circle has a diameter of 2.2 meters. The blade is tipped up 3 cm. The fan makes one rotation every second and is rated to move 80 m^3 of air per min. Answer the following questions to get a handle on the problem.

A. What is the circumference of the circle the outer edge of the fan blades make?
B. What is the difference in the speeds of the inner end of the blade and the outer end? Give your answer in meters per second.
C. What is the area of the ring made by the blades of the fan?
D. Find the effective radius of the fan. The effective radius of the fan is the radius where the area swept out by the blades is cut in half.
E. What is the pitch of the blade?
F. At what speed is the fan blade trying to push/pull the air through the fan at the effective radius?
G. What would the rise have to be at the outer edge of the blade to create the same rate of flow of air through the fan?
H. What would the rise have to be at the inner edge for the same rate?
I. If the fan blade was twisted so that the pitch was the values for parts G and H, what volume of air would the fan theoretically move? Give the answer in cubic meters per minute.
J. The manufacturer has rated the fan less than the actual amount of air calculated in part I. Besides being conservative determine at least two significant factors that cause part I to overrate the effectiveness of the fan.

Be sure to show all of your work. Organize and label the sections of your work including your data and diagrams.

Scoring Rubric Summarized

Solve Problems using – Measurement – Algebraic Sense – Number Sense – Geometric Sense

A. 2 points – Show how you got the outer circumference
B. 3 points – Show how you got the difference in speeds
C. 2 points – Show how you got the area of the ring
D. 3 points – Show how you got the effective radius
E. 2 points – Show how you found the pitch
F. 3 points – Show how you calculated the speed of the air
G. 2 points – Show how you calculated the rise at the outer edge
H. 2 points - Show how you calculated the rise at the inner edge
I. 3 points – Show how you calculated the rate at which air is moved
J. 2 points – Show understanding of the physical limitations of the calculations

5 points - Present work in an organized, clear, and logical manner, label appropriately, and use diagrams, mathematical language and notation.

Team Problem Answer Sheet

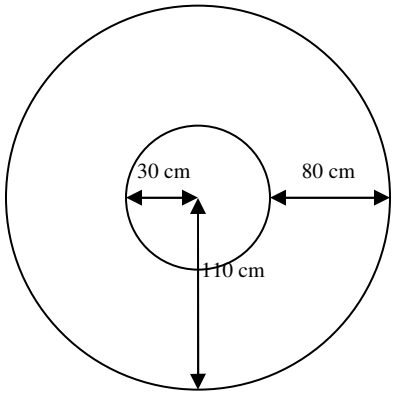
Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

Names

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

Sample Solution



A. Circumference is πD or 220π cm or 691 cm

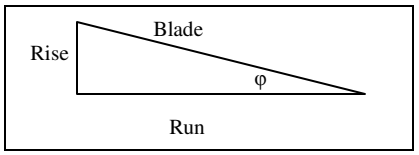
B. Speed is circumference divided by the time.
At the outer edge: 220π cm/s or 691 cm/s
At the inner edge: 60π cm/s or 188 cm/s
The difference is: 160π cm/s or 503 cm/s
When converted to m/s, 1.6π m/s or 5.03 m/s

C. Area of the ring: either $\pi(110^2 - 30^2)$ or $110^2\pi - 30^2\pi$ which calculates to be 11200π sq cm or 35200 sq cm but in square meters is 1.12π m^2 or 3.52 m^2.

D. The effective radius R:
$\pi(R^2 - 30^2) = (11200\pi)/2$ so $R^2 = 5600+900 = 6500$ and R = 80.6 cm.

E. The pitch: $3/(25^2 - 3^2) = 3/24.8 = 0.12$ with no units.

F. Air flow: the speed of the fan at a radius of 80.6 is $2\pi(80.6) = 507$cm/s. This divided by the run of the blade gives the number of 3 cm the air must move per second. Multiplying that answer by 3 cm gives the speed/flow of air in cm/s. This would be the same as multiplying the speed by the pitch: 507(0.12) = 61 cm/s.



G. At the outer edge to get the same flow the speed times the pitch (P) needs to be 61 cm/s. So 691 P = 61. P = 0.088. The tangent of φ is the pitch and the blade width times the sine of φ would give the rise. So φ = 5.04 degrees and the rise is 25 cm sin (φ) = 2.2 cm. This result can also be arrived at by letting the rise = x and setting up the ratio $0.088 = x/(25^2 - x^2)^{0.5}$ and solving for x.

H. Using the same procedure as in part G, the pitch at the inner edge is 0.32, φ is 18 degrees and the rise is 7.7 cm.

I. If you use the entire area of the fan in meters and the flow rate of 0.61 m/s the volume of air the fan is trying to move is $1.10^2\pi$ m^2 (0.61 m/s) = 2.32 m^3/s or 140 m^3/min.
If only the area of the ring is used, the volume is 3.52 m^2 (0.61 m/s) (60 s/min) = 129 m^3/min.

J. In either case, there will be significant air friction with the surrounding air causing eddies and reducing the flow. Also, the motor's physical presence tends to restrict the flow. The shape of and the obstacles in the room are also factors.