

WSMC High School State Competition

Planet Qubette

Team Problem

April 21, 2007

Picture a spherical planet that has an imaginary cube inscribed in it. This 'planet', Qubette, has 8 major cities located at the vertices of the imaginary cube. These cities are creatively named A, B, C, D, E, F, G, and H. By law, each power line and inter-city highway follows a path on the surface of Qubette corresponding to the edge of the cube that would be beneath. This allows the planet to maintain its open space, wildlife habitat, farmlands, etc. All inter-city highways have been built. There are no power lines between cities.

City	Population
A	30,000
B	50,000
C	100,000
D	300,000
E	20,000
F	50,000
G	100,000
H	200,000

This is a partial list of directly connected cities: A is connected to D and F; B to E; C to F and G; and D to E and C. The distance between connected cities is 10 quibbetts. The table gives the population of each city. Each city has enough power supply to run a city twice its size.

- Which city is the farthest from A, by highway? Show diagram to support your answer.
- A resident of one of the cities decided to visit each city and drive each highway by car. How many cities would need to be visited twice? Explain. What is the minimum distance the resident would have to travel to complete the trip **and return home**?
- What is the radius of the planet?
- If each city's power supply has a 5% chance of failure in a week, what is the probability that a city's power supply and at least two of the three neighboring cities (10 quibbetts away) have a failure in a single week?
- Assume that it costs \$10,000 to run a single power line between two cities to serve a maximum of 10,000 people, \$20,000 to run a line to serve 50,000 people, and \$30,000 for 100,000 people. The cities have decided to run just enough lines so that if any one city lost power, they could draw from other cities' reserves to maintain operation without forcing any city to reduce use. Determine a power network that meets the criteria at a minimum cost. State the cost and describe the network in words, symbols and/or diagrams.
- Qubette's gravity is 1/20 of the earth's gravity. Given, the density of Qubette is the same as the earth's density. Show the radius of Qubette is 1/20 of the earth's radius.
Using these given relationships:
 - Acceleration_{due to gravity} = Gravitational constant x mass / radius²
 - When the density is the same, the mass is proportional to the volume of the planet.
 - Volume_{of a sphere} = (4/3)π r³.

Show all of your work. Organize and label the sections of your work including your data and diagrams.

Scoring Rubric Summarized

Solve Problems – Reason Logically – Communicate Understanding – Make Connections
Using

Reasoning — Measurement – Algebraic Sense – Geometric Sense – Probability

- 2 points - Show how you determine which city is farthest from A.
- 6 point - Show how you determine cities visited twice and the distance traveled.
- 5 points - Show how you determine the radius of the planet Qubette.
- 5 points - Show how you determine probabilities of simultaneous power failures.
- 5 points - Show how you determine layout and cost of the power network.
- 3 points - Show how you show gravity is proportional to radius.
3 points - Overall communication.

Planet Qubette - Rubric for Scorers School _____

Team Members _____

Note: If a team is able to get the correct results for a section through some other correct means, they should earn equivalent points. There is rounding throughout and reasonable answers are acceptable. If a value(s) is calculated incorrectly and subsequent calculations based on the incorrect value are calculated appropriately (but of course the answer is incorrect) points for the calculations should be awarded.

A. Farthest from city A (2 points)

- | | | |
|--|--------------------------|--|
| | Reasons Logically | 1 pt – Determines G is the farthest away |
| | | 1 pt – Show labeled diagram to support answer. |

B. Travels all of the highways (6 points)

- | | | |
|--|--------------------------|---|
| | Solves Problems | 2 pts – Determines the number of total roads/distance (160 q) |
| | Reasons Logically | 1 pt – Labels distance in quibbetts |
| | Measurement | 2 pts – Determines that each city must be visited exactly twice |
| | | 1 pt – Explains reasoning why each city must be visited twice |

C. Radius of Qubette (5 points)

- | | | |
|--|------------------------|--|
| | Solves Problems | 3 pts – Demonstrates a strategy (understanding of the helpful geometry) for finding the radius |
| | Geometric Sense | |
| | Measurement | 1 pt – Determines the radius (approximately 8) |
| | | 1 pt – Uses the correct units (quibbetts) |

D. Probability of simultaneous power failure (5 points)

- | | | |
|--|--------------------|---|
| | Probability | 3 pts – Determines the probability of two neighbors have failure (0.036%) |
| | | 1 pt – Determines the probability that all three neighbors have failure (0.0006%) |
| | | 1 pt – Determines the probability that at least two have failure (0.037%) |

E. Determines a power network (5 points)

- | | | |
|--|------------------------|--|
| | Solves Problems | 2 pts – Determines a network that meets the power criteria |
| | Communication | 2 pts – Determines a minimum cost network ($\leq \$250,000$ is 1 point, $\leq \$230,000$ is 2 points) |
| | | 1 pt – Using words, symbols, communicates how they determined their answer. |

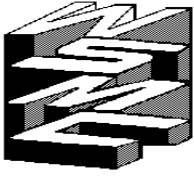
F. Verifies that gravity is proportional to the radius (3 points)

- | | | |
|--|--------------------------|---|
| | Solves Problems | 3 pts – Demonstrates or explains why the gravity is proportional to the radius using words, symbols, or a diagram |
| | Algebraic Sense | |
| | Makes Connections | |

Overall (3 points)

- | | | |
|--|----------------------|---|
| | Communication | 1 pt – Work is organized, labeled, and easy to follow |
| | | 1 pt – Uses diagrams to help explain/clarify the work |
| | | 1 pt – Uses appropriate mathematical language and/or notation |

	Total Points	29 points possible
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Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name _____ Team Number _____

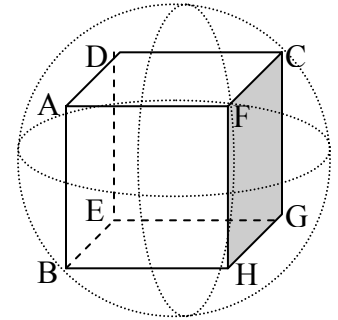
Names _____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored.

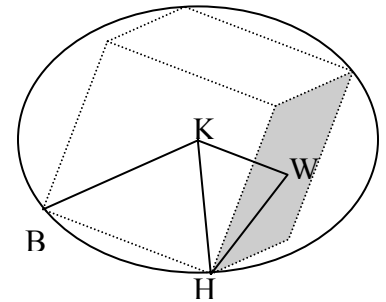
Sample Solution

A. The city farthest from A is G.

B. Since each city has three roads to it, to be able to travel every road, there would have to be two trips to each city. With two trips, that would require traveling four roads twice. There are many possibilities for this but the total trip would be 160 quibbetts.

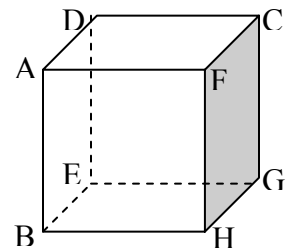


C. Draw the triangle from the center of the planet (K) to two cities like B and H. Draw segment KW where W is the center of a face of the imaginary cube. Draw segment WK. $\angle W$ is a right angle. If BH is x units, HW is $x(\sqrt{2})/2$ and KW is $x/2$. $\tan(\angle WHK) = (x/2) / [x(\sqrt{2})/2] = 1/(\sqrt{2})$. Using \tan^{-1} we find $\angle WHK = 35.26^\circ$. That makes $\angle KHB$ and $\angle KBH$ each have to be 54.74° . Therefore $\angle KHB = 180^\circ - 2(54.74^\circ) = 70.53^\circ$. Consequently, the length of arc BH = $(70.53^\circ/360^\circ) 2\pi (KH)$. Since the length of arc BH is 10 quibbetts, solving this equation for KH gives a planet radius of 8.12 quibbetts.

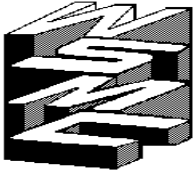


D. The probability that exactly two of the three neighbors have power failure as well is $P(\text{failure of home city}) \cdot (\text{ways to pick two of three cities}) \cdot P(\text{a city's power fails}) \cdot P(\text{a city's power fails}) \cdot P(\text{city's power doesn't fail}) = (0.05) \cdot ({}_3C_2) \cdot (0.05)^2 \cdot (0.95) = 0.036\%$ chance.
The probability of a home city and all three neighbors failing is $(0.05)^4 = 0.0006\%$. Therefore there is a 0.037% chance of at least two neighbors losing power when a home city does.

E. A possible solution: two 100k lines C to D; one 100k line from C to G, G to H, and D to E; one 50k line from A to D, B to E, B to H, and F to H. Total cost is $5(\$30,000) + 4(\$20,000) = \$230,000$.



F. Since gravity is proportional to mass divided by the radius squared and mass is proportional to the radius cubed, gravity must be proportional to radius cubed divided by radius squared or simplified just the radius. Therefore, since there is 1/20 the gravity, there must be 1/20 of the radius.



Team Problem Answer Sheet

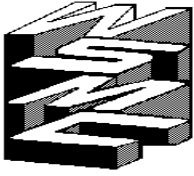
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Names _____

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A large grid of 20 columns and 30 rows of small dots for writing answers.



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Name _____	School _____	Grade _____
Name _____	School _____	Grade _____
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