

The Lot Problem

Team Problem

April 22, 2006

The map shows a quadrilateral shaped piece of undeveloped lakefront property that the owners are trying to decide whether to divide into two pieces before they sell it or to leave it as is. The first table shows the history of the sales of comparable property at the same lake. The second table shows the relative value of the dollar over the period of sales of the sample properties.

1. Calculate the values of the comparable properties in **year 2000 dollars**.
2. The owners may decide to divide their lot into two pieces with one straight line yielding two lots of equal area and equal lake frontage (length of the shore line). Determine the area and lengths (to the nearest hundredth of a foot) of **all property lines** of the two lots.
3. Lot prices are based on value per lake-frontage foot and value per square foot where larger parcels have a different value per square foot than small parcels. Estimate the **sale price of the lot as is** and the **price of each of the two potential lots**. Estimates should be based on year 2000 dollars.
4. The cost to divide the lot into two pieces is estimated to be \$8000. **Determine** whether it would be financially advisable to divide the original lot before the owners sell the property. Make a recommendation with support.

Formulas:

$$\text{Area}_\Delta = 0.5 \cdot \text{side} \cdot \text{side} \cdot \sin(\text{angle between these sides})$$

$$\text{Law of Sines: } \sin(A)/a = \sin(B)/b = \sin(C)/c$$

$$\text{Law of Cosines: } c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$

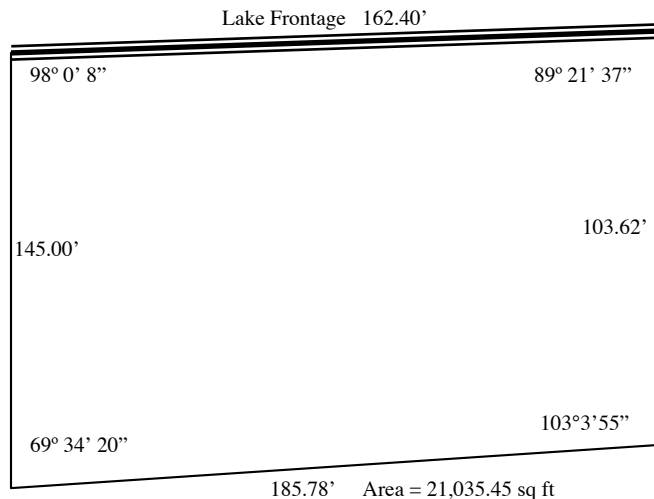
Be sure to show all of your work. Organize and label the sections of your work including your data and diagrams.

Scoring Rubric Summarized

Solve Problems using – Measurement – Algebraic Sense – Number Sense – Geometric Sense – Communication

- ✓ 6 points - Show how you got the 2000 dollars values.
- ✓ 10 points - Show how you divided the lot.
- ✓ 7 points - Show how you determined the values of the lot options.
- ✓ 3 points - Show how you determined whether the owners should divide the lot.

Communication - 3 points - Present work in an organized, clear, and logical manner, label appropriately, and use mathematical language and notation.



Comparable Parcels Sales History				
Lot	Area (Square feet)	Lake Frontage (Feet)	Date of Sale	Sale Price of Parcel
A	22000	175	1975	47,250
B	12000	75	1982	64,583
C	24000	160	1988	108,696
D	20000	160	1988	93,478
E	10000	75	1995	85,870
F	11000	75	1998	101,415

Value in Year 2000 Dollars of a Prior Year's Dollar (Assume linear change between the years listed.)							
Year	1970	1975	1980	1985	1990	1995	2000
Value of dollar	4.00	3.00	2.00	1.50	1.30	1.15	1.00

The Lot Problem - Rubric for Scorers Team # _____ School _____

Note: If a team is able to get the correct results for a section through some other correct means, they should earn equivalent points. There is rounding throughout and reasonable answers are acceptable. If a value(s) is calculated incorrectly and subsequent calculations based on the incorrect value are calculated appropriately (but of course the answer is incorrect) points for the calculations should be awarded.

1. Values in 2000's Dollars (6 points)

	Number Sense	2 pt – Determined factors for unlisted years (2 pt for 3 or 4, 1 pt for 1 or 2)
	Algebraic Sense	3 pt – Determined values in year 2000 dollars (3 pt for 6 or 7, 2 pt for 4 or 5, 1 pt for 2 or 3)
	Communication	1 pt – Communicated their method for determining the values in year 2000 dollars

2. Dimensions of Divided Lot (10 points)

	Solves Problems:	1 pt – Determined lake frontage of each lot
	Measurement	1 pt – Determined the area of each lot
	Geometric Sense	1 pt – Communicated a viable procedure for finding dimensions of lower and center property lines
	Communication	3 pt – Determined the lower property line lengths (3 pt for both to hundredths +/- 0.10, 2 pt for both close* or one correct, 1 pt for one close)
		2 pt – Determined the length of the center line (2 pt for correct to hundredths, 1 pt for close*)
	2 pt – Labeled units on answers (2 pt for consistent use, 1 pt for occasional use)	
	*Scorers will determine their definition of close after reviewing several papers	

3. Determining the Sale Price of the Possible Parcels (7 points)

	Solves Problems:	1 pt – Communicated understanding that price per acre is different for small and large parcels
	Algebraic Sense	
	Measurement	1 pt – Determined the value of a square foot of the larger parcels (~\$5.25 / sq ft)
	Communication	1 pt – Determined the value of a square foot of the smaller parcels (~\$8.75 / sq ft)
		1 pt – Determined the value of a foot of lake frontage (~\$150 / ft)
		1 pt – Determined the value of the whole lot (~\$136,000)
		1 pt – Determined the value of each smaller lot (~\$105,000)
	1 pt – Communicated their method for finding the above	

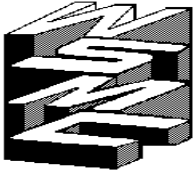
4. Determines Plan of Action for Owners (3 points)

	Reasoning	1 pt – Made a recommendation on the plan of action for the owners
		1 pt – The recommendation is supported with information including the \$8000 cost to divide the parcel
		1 pt – The recommendation is to divide the original property before selling

Overall (3 points)

	Communication:	1 pt – Is organized and easy to follow
		1 pt – Used diagrams and/or tables to help organize and clarify their work
		1 pt – Used appropriate mathematical language and/or notation

	Total Points	29 points possible
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Team Problem Answer Sheet

Only this page will be evaluated. You may use front side only. You might want to draft your answer on scratch paper first.

School Name **SAMPLE SOLUTION** Team Number

Names _____

Support all your work with clear and convincing information and calculations. Only answers on the front of this page will be scored. Special answer sheet for Junior varsity teams. Support all your work with clear and convincing information and calculations.

1. Year 2000's dollars values are in the table.

Similar Parcels Sales History					
Lot	Area (Square feet)	Lake Frontage (Feet)	Date of Sale	Sale Price of Parcel	2000 Dollars
A	22000	175	1975	47,250	$47,250 \times 3.00 = 141,752$
B	12000	75	1982	64,583	$64,583 \times 1.80 = 116,249$
C	24000	160	1988	108,696	$108,696 \times 1.38 = 150,000$
D	20000	160	1988	93,478	$93,478 \times 1.38 = 129,000$
E	10000	75	1995	85,870	$85,870 \times 1.15 = 98,751$
F	11000	75	1998	101,415	$101,415 \times 1.06 = 107,500$

2. Lake frontage will be $162.40' / 2 = 81.20'$

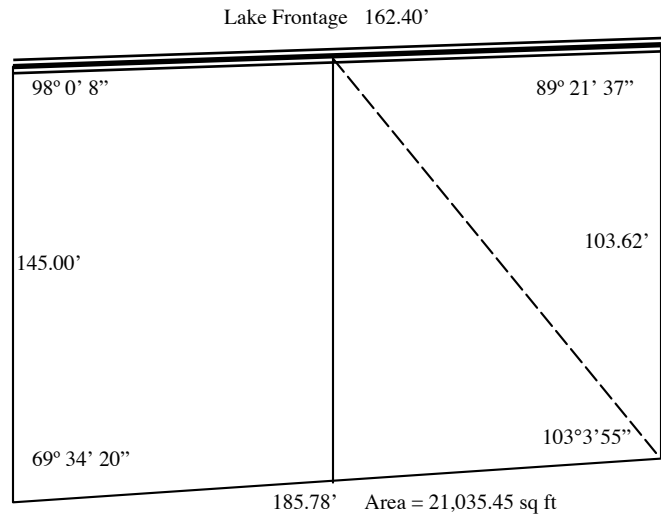
Area of upper right triangle = $.5 \times 81.20' \times 103.62' \times \sin(89^\circ 21' 37'') = 4206.710 \text{ sq ft}$

Area of adjacent triangle = $21035.45 / 2 - 4206.710 = 6311.015 \text{ sq ft}$

Length of dashed line = $(81.2^2 + 103.62^2 - 2 \times 81.2 \times 103.62 \times \cos(89^\circ 21' 37''))^{0.5} = 130.9299835'$

Lower angle in upper right triangle = $\cos^{-1}((130.93^2 + 103.62^2 - 81.2^2) / (2 \times 103.62 \times 130.93)) = 38.32636363 = 38^\circ 19' 35''$

Therefore the lower right angle of the lower triangle is $103^\circ 3' 55'' - 38^\circ 19' 35'' = 64^\circ 44' 20''$



Therefore the area of the lower triangle can be represented by: $6311.015 \text{ sq ft} = 0.5 \times 130.9299835' \times X \sin(64^\circ 44' 20'')$ where **X** is the lower side of the lower triangle. Solving for **X** yields $106.60'$ implying that the lower line for the left parcel is $185.78 - 106.60 = 79.18'$.

The line that divides the lots is: $(106.60^2 + 130.93^2 - 2 \times 106.60 \times 130.93 \times \cos(64^\circ 44' 20''))^{0.5} = 128.82'$

3. Subtracting the value of Lot D from Lot C yields \$21000. Dividing by their difference in area gives \$5.25 per square foot for larger lots. Using Lot D, $20000 \text{ sq ft} \times \$5.25/\text{sq ft} = \$105,000$. The frontage must be $(\$129,000 - \$105,000) / 160 \text{ ft} = \$150 / \text{ft}$. Therefore the smaller lot price must be (using Lot E) $(\$98,751 - 75 \text{ ft} \times \$150 / \text{ft}) / 10,000 \text{ sq ft} = \$8.75 / \text{sq ft}$. Therefore the value of the whole lot is $21035.45 \text{ sq ft} \times \$5.25 / \text{sq ft} + 162.40 \text{ ft} \times \$150 / \text{ft} = \$134,796$ and the value of each smaller lot would be $21035.45 \text{ sq ft} / 2 \times \$8.75 + 81.2 \text{ ft} \times \$150 / \text{ft} = \$104,210$.

4. $2 \times \$104,210 > \$134,796 + \$8000$ so the owners would benefit by dividing the lot.