



WSMC High School Regional Competition

Topical Problems

Part One

March 17, 2004

Directions: Mark on the answer form the letter that is closest to the correct answer. Make sure that your name(s) are on the answer form. If you are on a team put your team number and school name on the answer form. Remember that there are 5 points awarded for a correct response, 1 point for no response, and 0 points for an incorrect response.

In Euclidian Geometry, three distinct points that are non-collinear (don't lie in a straight line) determine a plane. That is to say that only one plane will contain all three points.

1. How many distinct planes are determined by four non-coplanar (don't lie in the same plane) points?
A. 2 B. 3 C. 4 D. 5
2. The planes determined by the four points in the previous problem divide space into how many different regions?
A. 14 B. 18 C. 9 D. 15
3. How many distinct planes are determined by sixteen points, no four of which are coplanar?
A. 560 B. 120 C. 420 D. 48

A rectangular lawn is being designed and the total amount of sod available for the lawn is 667 square meters. There is some indecision over what the dimensions of the lawn should be.

4. Ulysses believes in the ancient Greeks and wants the length to width ratio to be the Golden Ratio: $\frac{(1+\sqrt{5})}{2}$. If Ulysses gets his way, what would the length of the longest side in meters be?
A. 35 B. 21 C. 32 D. 26
5. The gardener knows the plan to put a four meter wide flower bed around the lawn and is tired of being over worked. She wants the flower bed to have a minimal area and she does not care about the Golden Ratio. If she gets her wish what is the area in square meters of the flower bed?
A. 320 B. 420 C. 480 D. 400
6. In the end, it was decided that the lawn had to fit the available rectangular space available. A four meter wide flower bed was planted around the 667 square meter rectangular lawn which used the entire 1215 square meters. What was the longest dimension of the lawn?
A. 50 B. 45 C. 40 D. 55

An automatic garage opener has 8 switches, numbered 1-8, that can be turned on or off in any combination to determine the frequency that it will respond to. The remote unit has an identical set of switches.

7. How many different frequencies can be selected?
A. 40,320 B. 250 C. 64 D. 16,770
8. If 5 nearby neighbors installed the same garage opener systems and randomly set their frequency controlling switches, what is the probability that at least two will have the same frequency?
A. 0.10 B. 0.04 C. 0.09 D. 0.01
9. How many units would have to be installed near one another for there to be a 50% chance that at least two will be using the same frequency?
A. 10 B. 15 C. 20 D. 30

In the pattern of numbers at the right each row is telling how many times each digit is repeated in the previous row. For example the third row says that there is 2 “1”s in the second row, 1 “2” in the second row, and 1 “3” in the second row.

1112
3112
211213
312213

10. What is the maximum number of rows there could have been in the pattern prior to the first row?
A. 1 B. 2 C. 3 D. 4
11. How many rows can be added without repetition?
A. 2 B. 5 C. 8 D. 11
12. Imagine starting the pattern with just the number 1 and going until there is repetition. Then imagine starting with the number 2 and going until repetition. Then the number three, then four, then five and then six. Each time notice the number that repeats. How many different numbers are among the repeated numbers?
A. 5 B. 3 C. 4 D. 6

An insurance company claims that in a state, customers that switch to it save on the average 15% on their insurance over a year's time. This ad has caused 100,000 customers to switch and try the company's insurance for one year.

13. The company claims that the average customer that switches in Washington saves \$178. What was the price of their policy before they switched?
A. \$27 B. \$205 C. \$900 D. \$1200
14. If 3 out of every 25 policy holders switched how many policy holders are there?
A. 300,000 B. 800,000 C. 3,000,000 D. 1,240,000
15. At the end of one year, 1 in 15 switches back to their original company. If each following year the same continues to happen: 3 out of 25 switch to the company and 1 out of 15 switch back and the total number of policies in the state remains constant, what percentage of the policies will this insurance company tend to carry over a long period of time?
A. 70% B. 50% C. 90% D. 30%



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The Cheops Pyramid in Egypt is a regular square based pyramid with a base edge of 230 meters. Its slant edge (distance from the top to a corner) is 219 meters.

1. What is the height in meters of the pyramid?
A. 150 B. 185 C. 320 D. 250
2. What is the measure of the angle in degrees between a face of the pyramid and the base (ground)?
A. 47 B. 58 C. 52 D. 61
3. If the pyramid is made of stone with a density of 12×10^3 kilograms per cubic meter, and the interior is 15% hollow, what is the mass in kilograms of the pyramid?
A. 3.1×10^{10} B. 4.7×10^9 C. 7.9×10^{10} D. 2.6×10^{10}

An earlier survey was conducted on patients at a nearby hospital over a one month period. In that survey exactly one-fourth of the patients suffered from high blood pressure, exactly one-sixth had hypertension, and exactly one-third had high cholesterol. Assume each of these ailments do not depend on each other.

4. If a sample of 864 patients is selected, how many patients in this sample probably have hypertension but do not have high cholesterol?
A. 270 B. 180 C. 210 D. 240
5. In the same 864 patients sampled, how many probably didn't suffer from any of these three conditions?
A. 215 B. 245 C. 275 D. 335
6. What is the minimum number of patients that could have been in the original survey?
A. 36 B. 12 C. 144 D. 72

In a democratic country, there are several candidates running for an office. Therefore the voters are allowed to have two votes. In this particular election, candidates A, B, C, D, and E are running for election. The end results were reported in this way:

- Half of the eligible voters votes for A and their second vote was split among B, C, D, and, E in proportion of 4:3:2:1 respectively.
- Half of the remaining voters voted for B and their second vote was split among C, D, and E in the proportion of 3:1:1 respectively.
- Two thirds of the remaining voters voted for both C and D.
- 540 eligible voters did not vote.

7. How many voters were there?
A. 4000 B. 6000 C. 8000 D. 10000
8. Which arrangement shows correctly the ranking of the candidates from the most votes received to the least received?
A. ABCDE B. ABDCE C. ACBDE D. ACDBE

In the 3000 mile long Race Across America, riders will do most everything on their bike and keep off-bike time to a minimum. Because the race clock starts at the starting line in San Diego and doesn't stop for a rider until he/she crosses the finish line in Atlantic City. A Washingtonian, Allen Larsen, won the race in 2003 in just under 9 days (8days, 23 hours and 36 minutes).

9. If Allen limited his off-bike time to 2.00 hours in every 24 hours, what would his on the bike average speed in miles per hour have to be to complete the race in 9 days?
 - A. 16.8
 - B. 15.3
 - C. 14.5
 - D. 15.8
10. In the 2004 race, Allen would like to break the record by having an overall average speed (including off-bike times) of 15.2 miles per hour. How long in days and hours would it take him to complete the race?
 - A. 8 days 5 hours
 - B. 7 days 23 hours
 - C. 8 days 15 hours
 - D. 8 days 11 hours
11. Given that Allen's on-bike daily speed average decreases at a rate of 0.5 miles per hour daily because of fatigue and he plans to be off the bike exactly two hours in every twenty four hour period, what will he have to have for an on-bike average in miles per hour on the first day if he is to finish in 8 days?
 - A. 17.5
 - B. 22
 - C. 19
 - D. 20.5

In an optics experiment, the data in the table was collected. Using the function that fits the data very well, (and there is one!), answer the following questions.

X	Y
5.2	17.3
7.1	9.2
8.9	7.2
4.8	24.0
6.7	9.9
10.0	6.7
11.9	6.0
15.3	5.4
-6.5	2.5
-8.6	2.7
-2.5	1.5
-15.1	3.2

12. What is the value of Y when X=5.7?
 - A. 15.8
 - B. 13.6
 - C. 12.4
 - D. 14.7
13. What is the value of Y when X=2?
 - A. -12
 - B. 210
 - C. -6
 - D. 11

A bi-metal strip is a strip of two laminated layers of different metals. In this case, the top layer is 0.500 mm. thick and made of aluminum and the bottom layer is 0.500 mm. thick and made of steel. The length of the strip is 5.000 cm. at room temperature (21 degrees Celsius). When heated the bi-metal strip will bend because one of the metal layers expands more than the other.



The formula for linear expansion of metal is given by the following formula $\Delta l = kL \Delta T$ where Δl is the change in length, k is the expansion constant for the metal, L is the length of the metal at a starting temperature in degrees centigrade, and ΔT is the change in temperature in degrees centigrade. The expansion constant for steel is 10.5×10^{-6} and for aluminum is 23.8×10^{-6} .

14. How much in millimeters does the length of the steel layer increase when heated to 100 degrees Celsius?
 - A. 9.4×10^{-3}
 - B. 9.4×10^{-2}
 - C. 4.1×10^{-3}
 - D. 4.1×10^{-2}
15. What is the radius of curvature in millimeters of the bi-metal strip when heated to 100 degrees Celsius?
 - A. 5000
 - B. 50
 - C. 500
 - D. 5

Answer Key Regional Topical March 17, 2004

Part One		Part Two	
1.	C 4	1.	A $146 \frac{2}{3}$
2.	D 15	2.	C 51.8°
3.	A 560	3.	D 2.6×10^{10}
4.	C 32.85	4.	B 96
5.	C 477	5.	D 360
6.	B 46	6.	D 72
7.	B 256	7.	B 6480
8.	A 0.105	8.	C ACBDE
9.	C 19 is 49.5%, 20 is 53.3%	9.	B 15.2
10.	C 1, 11, 21	10.	A 8 days 5 hours
11.	B 6	11.	C 19
12.	B 1-4 give same, 5&6 are different	12.	B 13.4
13.	D \$1186.67	13.	A -12
14.	B $833333 \frac{1}{3}$	14.	D 4.1×10^{-2}
15.	A 64%	15.	C 472